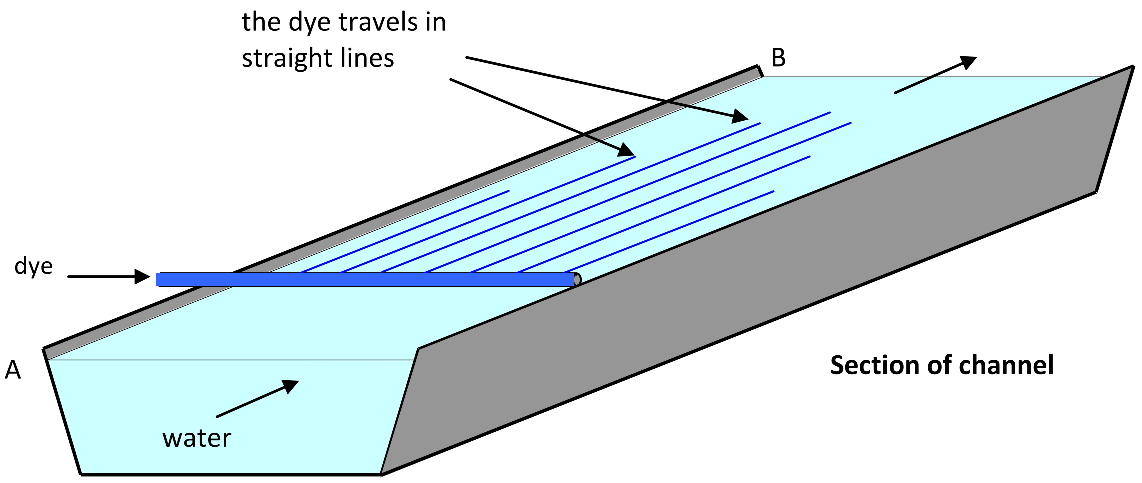
In this activity you will use functions to model data.
You will also calculate percentage errors.

Information sheet

In a laboratory experiment to study the flow of water along an open channel, dye is released into the stream at regular intervals across its width. When the water is pumped into the channel at a relatively low rate, the flow is steady and the dye travels along the channel in straight lines as shown in the diagram.
If a photograph is taken a short time after the dye is released, the distance travelled, and hence the velocity of the stream, can be determined.



Section of channel

The results of two such experiments are given in the table below.

Some of the values from the second experiment have been lost.

Perpendicular distance from edge of channel (AB) x metres	**Velocity of water** u metres per second	
	Experiment 1	Experiment 2
0	0	0
0.1	0.12	0.09
0.2	0.21	
0.3	0.26	
0.4	0.28	0.21
0.5	0.26	
0.6	0.21	
0.7	0.12	0.09
0.8	0	0

Think about…

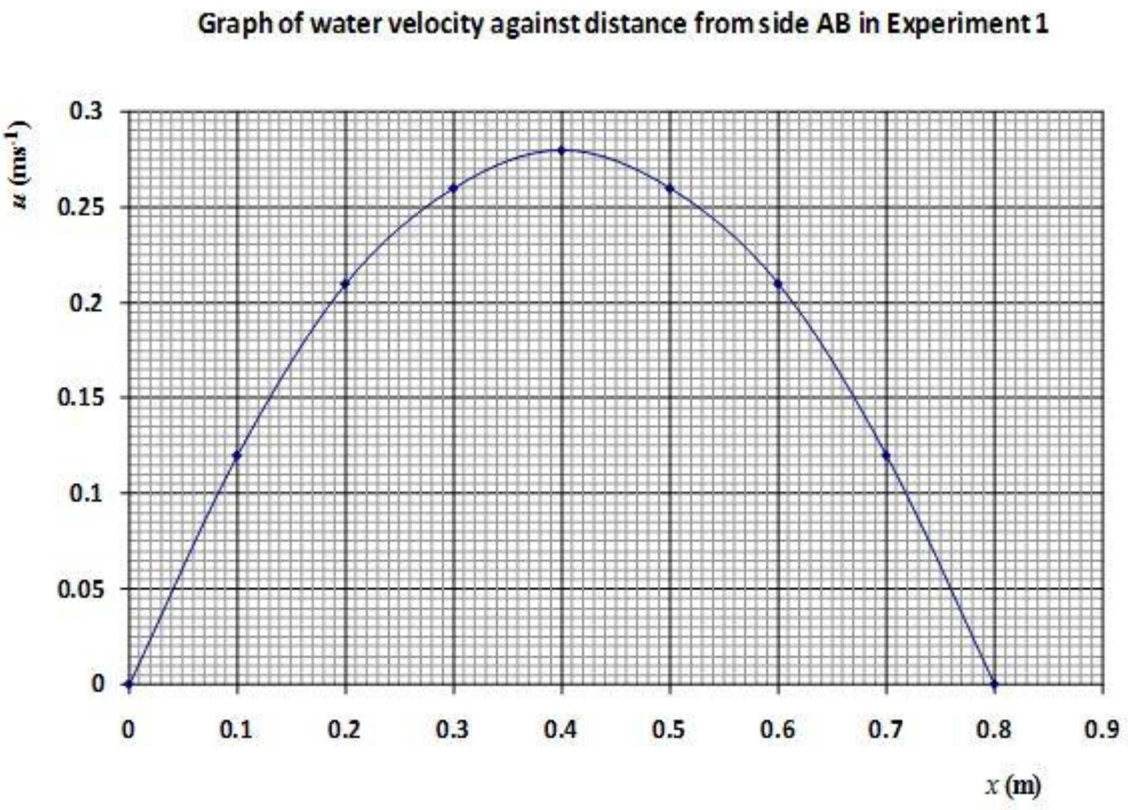
Look at the table on the previous page. Where in the channel is the water flowing most quickly?

Look at the diagram and the data. What type of function is likely to provide a good model of velocity against distance?

How do you find the percentage error if the model does not match the data?

Try these

The diagram below shows a graph of the velocity of the water, u metres per second, plotted against distance from the side of the channel, x metres, in the first experiment. The data points have been joined by a smooth curve.



a Explain clearly how the values of u at the points where the curve meets the x axis, and at the turning point, relate to the real situation.

b The part of the curve for $0 \leq x \leq 0.2$ can be modelled by the linear function which passes through the points (0, 0) and (0.2, 0.21).

i Find the equation of this function, giving your answer in the form $u = mx + c$.

ii Calculate the error between the value of u predicted by this linear function when $x = 0.1$ and the actual velocity.

iii Express this error as a percentage of the actual velocity.

c The complete curve can be modelled by a quadratic function of the form $u = ax(b - x)$ where a and b are constants.

i Find the values of a and b which will give a quadratic model with the same intercepts on the x axis, and the same turning point as the curve drawn using the original data.

ii Calculate the error between the value of u predicted by the quadratic function when $x = 0.1$, and the actual velocity.

iii Express this error as a percentage of the actual velocity.

d The rate at which the water is pumped along the channel was changed for the second experiment, so that the maximum velocity was reduced from 0.28 metres per second to 0.21 metres per second.

i Find the percentage reduction in the maximum velocity.

ii Assuming that the velocity of the water is reduced by the same proportion at all points across the channel, calculate the experimental values which were lost, giving your answers to two decimal places.

e In a third experiment, the rate at which water is pumped along the channel is increased until the maximum velocity of the water is 0.32 metres per second. The velocity at the sides remains zero.

i Describe the geometrical transformation which could be applied to the graph shown to give a maximum velocity of 0.32 metres per second.

ii Find the quadratic function that models the velocity for the new rate of flow.

iii Use the function you found in part **e ii** to predict the velocity of the water at points in the channel where $x = 0.2$.

Reflect on your work

- Describe how you found a linear function and a quadratic function to model the data.
- Compare your method with those used by other students.
  Which methods do you prefer?
- Explain how to find the percentage error when the data and model do not agree.